Math 109 Midterm 2 Practice

Print Name: ______________________________ **Section:** ________________

JHEDID: ______________________________

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature:________________________________ **Date:** __________________

- This is a 50 minute **closed book** exam. **No notes, books, or calculators are allowed.**
- Present your solution to each problem in a clear and orderly fashion. **Show all your work.** An answer without justification will not receive full credit.
- This exam contains 6 pages (including this cover page) and 5 questions. The last page is intended for use as scrap paper.

The table on the right is for grading purposes. Please do not write in it.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (20 points) Find the solution to the following initial value problem

$$y' = e^{x-y}; \quad y(3) = 5$$

Points earned: ______ out of 20 points

2. (20 points) Find the solution to the following initial value problem

$$xy' + 2y = x^2; \quad y(1) = 1$$

Points earned: ______ out of 20 points

3. Consider the following parametric curve

$$x(t) = t^2 + 1; \quad y(t) = t^3 - 1$$

   (a) (10 points) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of the parameter t.

   (b) (10 points) Sketch the curve labeling all x and y intercepts.

Points earned: _____ out of 20 points

4. Consider the curve given by the polar equation

$$r = \theta^2 - \pi\theta$$

   (a) (10 points) Sketch the curve.

   (b) (10 points) Find the area enclosed by the inner loop of the curve.

 Points earned: _____ out of 20 points

5. (20 points) Determine whether the sequence $\{\frac{n^2}{n!}\}_{n=1}^{\infty}$ converges, and if so, find the limit.

 Points earned: _____ out of 20 points

These pages are intended for use as scrap paper.

Trigonometric Identities

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

- Sum and Difference Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

- Double Angle Formulas

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

- Half Angle Formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

- Product Formulas

$$\sin A \sin B = \frac{1}{2}\left(\cos(A - B) - \cos(A + B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A - B) + \cos(A + B)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A + B) + \sin(A - B)\right)$$

Differentiation Formulas

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{x^2 + 1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

Integration Formulas

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \cot x dx = -\ln|\csc x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

Trigonometric Substitution

$$\sqrt{a^2 - x^2} \Longrightarrow x = a \sin\theta$$

$$\sqrt{a^2 + x^2} \Longrightarrow x = a \tan\theta$$

$$\sqrt{x^2 - a^2} \Longrightarrow x = a \sec\theta$$